

Comment on “Phase and Phase Diffusion of a Split Bose-Einstein Condensate”

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Recently Javanainen and Wilkens [1] (hereafter JW) analysed an experiment in which an interacting Bose condensate, after being allowed to form in a single potential well, is “cut” by splitting the well adiabatically with a very high potential barrier. They found that following the cut the rate R at which the two halves of the condensate lose the “memory” of their relative phase is $\sqrt{N}\xi$, where $\xi \equiv d^2E/dk^2$, k being the number of particles transferred between the two halves of the split trap ($\hbar = 1$). We show here that the problem posed by JW reduces to one already studied in the literature [2–4], and that the true value of R is much smaller.

We interpret JW’s “adiabatic condition” as the statement that the quantity $\lambda \equiv (dV/dt)/\hbar\omega_0^2$ is $\ll 1$, where V is the barrier height and ω_0 the small-oscillation frequency in the unsplit well, while simultaneously $r \equiv \lambda\omega_0 \gg R$ where R is the phase diffusion rate which we will eventually calculate. We shall assume the condition (experimentally realistic for the BEC alkali gases and implicit in JW’s Eq. (11)) $Q \equiv (15Na/l)^{1/5} \gg 1$ (with a and l defined as in JW).

Let us consider a particular value of the barrier height V_0 large enough that the contact between the two halves of the well is by Josephson tunnelling. We define the Josephson coupling energy E_J and the Josephson plasma frequency $\omega_p \equiv (E_J\xi)^{1/2}$ in the standard way; we remark that, for the BEC alkali gases, the conditions $\omega_0 \gg \omega_p \gg R$ and $E_J \gg \xi$ can be simultaneously satisfied over a wide range of barrier heights [5].

We now first consider a rather artificial two-stage process. At stage I we raise the barrier from zero to V_0 , at a rate r such that $R \ll r \ll \omega_p$. Because of the second inequality, this process is indeed strictly “adiabatic” and the final state of the system is with negligible error the groundstate of the (interacting) Hamiltonian for the final V_0 . This in turn can be well approximated [5] by the standard Josephson “pendulum” Hamiltonian (see e.g. Eq. (1) of Ref. [4]). The crucial point to note is that the fluctuations, in the groundstate [6] of this Hamiltonian, of the relative particle number k are by no means of order \sqrt{N} but rather much smaller, of order $(E_J/\xi)^{1/4}$. The phase fluctuations, while finite, are correspondingly of order $(\xi/E_J)^{1/4} \ll 1$. Note that the argument of this paragraph does not depend on the (incorrect) assumption

that Eq. (1) of Ref. [4] describes the unsplit well.

At stage II we raise the barrier height from V_0 to “infinity” at a rate r' such that $\omega_p \ll r' \ll \omega_0$, and then leave the system alone for a time $t..$. The problem of phase diffusion at this stage is precisely that considered in Refs. [2–4]; from Eq. (19) of Ref. [3] we find that the dephasing rate R is approximately

$$R = (E_J\xi^3)^{1/4}. \quad (1)$$

We now turn to the more “natural” case of a single smooth variation of the barrier height with time. It is clear that we should expect a formula of the general form (1) still to apply, provided that E_J is taken to be of the order of the Josephson energy at the point where the adiabatic condition is first violated (i.e. when $|d\omega_p/dt| \sim \omega_p^2$), which (if $\lambda \ll 1$) can be shown to lie well within the tunnelling regime. Thus, the true dephasing rate in the experiment analysed by JW is always smaller than the result they give by a factor of order $N^{-1/2}(E_J/\xi)^{1/4}$. Using the fact [5] that $E_J \propto \exp(-S_0)$ and that the WKB exponent S_0 is a slowly varying factor times V_0/ω_0 , it is straightforward to show that up to factors of order $|\ln \lambda|$ this ratio is of order $\sqrt{\lambda}/Q$. We conclude that, if $Q \gg 1$, JW’s result cannot describe correctly an adiabatic ($\lambda \ll 1$) separation process.

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- [6] Which is of course a superposition of Hartree-Fock states corresponding to (slightly) different values of the relative phase, see Eqs. (15) and (18) of Ref. [3] and the intervening discussion.